Sistemas lineales

Question 1

Top 1 Bottom Focus Help

Calcular el vector $\mathbf{x}^{(k)}$ de la iteración k-ésima cuando k=5 si se utiliza el método de Jacobi en la resolución del sistema:

$$\begin{bmatrix} -7 & -4 & -1 & -1 \\ -1 & 8 & -5 & 0 \\ -1 & -2 & -4 & 0 \\ 0 & 2 & 0 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -10 \\ 35 \\ 3 \\ -36 \end{bmatrix}, \quad \text{tomando } \mathbf{x}^{(0)} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

que tiene como solución el vector $\mathbf{x} = [-1,3,-2,7]^T$. Calcular en cada iteración k el valor de la estimación del error absoluto $||\mathbf{e}^{(k)}|| = ||\, \mathbf{x}^{(k)} - \mathbf{x}^{(k-1)}\, ||_\infty$ y también del error relativo $||\, \mathbf{x}^{(k)} - \mathbf{x}^{(k-1)}\, ||_\infty \,/\, ||\, \mathbf{x}^{(k)}\, ||_\infty$. Dar los resultados con cuatro decimales exactos.
You have not attempted this yet

The teacher's answer was:

$$\begin{bmatrix} -0.883 & 3.43 & -1.99 & 6.91 & 0.701 \end{bmatrix}$$

Solution:

A continuación aparece el ejercicio resuelto por los diferentes métodos, unas 20 iteraciones con cada método. Las diferencias en la velocidad de convergencia pueden ser importantes.
En el caso de Jacobi, la componente i-ésima de la iteración (k+1)-ésima se obtiene utilizando la fórmula:

$$x_i^{(k+1)} = \frac{1}{a_{ii}} \left(b_i - \sum_{j=1}^{i-1} a_{ij} x_j^{(k)} - \sum_{j=i+1}^{n} a_{ij} x_j^{(k)} \right)$$

para cada componente i = 1, 2, ... , n de cada iteración $\mathbf{x}^{(1)}$, $\mathbf{x}^{(2)}$, ..., cuando k = 0, 1, 2, ...

JACOBI						
Iter.	$\mathbf{x}^{(k)}$				Estimación errores	
k	$x_1^{(k)}$	$x_2^{(k)}$	$x_3^{(k)}$	$x_4^{(k)}$	$\|\| \mathbf{e}^{(k)} \|\|_\infty$	$\|\| \mathbf{e}^{(k)} \|\|_\infty / \|\| \mathbf{x}^{(k)} \|\|_\infty$
0	0	0	0	0	0	0
1	1.428571429	4.375000000	-.7500000000	6.000000000	6.000000000	1.000000000
2	-1.821428571	4.084821429	-3.294642858	7.458333333	3.250000000	.4357541900
3	-1.500425170	2.088169642	-2.337053572	7.361607143	1.996651787	.2712249850
4	-.4824617343	2.726788371	-1.418978528	6.696056547	1.017963436	.1520243189
5	-.8834616429	3.427830704	-1.992778752	6.908929457	.701042333	.1014690246
6	-1.232496219	3.019080575	-2.243049941	7.142610235	.408750129	.5722699623e-1
7	-.9965546557	2.819031760	-1.951416233	7.006360192	.291633708	.4162413864e-1
8	-.9044387143	3.030795522	-1.910377216	6.939677253	.211763762	.3051492948e-1
9	-1.021783160	3.067959401	-2.039288082	7.010265173	.128910866	.1838887158e-1
10	-1.034687813	2.972722054	-2.028533910	7.022653133	.95237347e-1	.1356144824e-1
11	-.9835724914	2.977830330	-1.977689074	6.990907352	.511153216e-1	.7311686313e-2
12	-.9892199429	3.015997768	-1.993022042	6.992610110	.38167438e-1	.5458253413e-2
13	-1.009082734	3.005708731	-2.010693898	7.005332590	.198627911e-1	.2835381596e-2
14	-1.002496230	2.992180972	-2.000583682	7.001902910	.13527759e-1	.1932011794e-2
15	-.9957204457	2.999323170	-1.995466428	6.997393657	.7142198e-2	.1020694040e-2
16	-.9998885586	3.003368426	-2.000731474	6.999774390	.5265046e-2	.7521736711e-3
17	-1.001788089	2.999556759	-2.001712073	7.001122808	.3811667e-2	.5444365289e-3

18	-.9996625400	2.998706444	-1.999331357	6.999852253	.2380716e-2	.3401094643e-3
19	-.9993352400	3.000460085	-1.999437587	6.999568815	.1753641e-2	.2505355753e-3
20	-1.000281651	3.000434602	-2.000396232	7.000153362	.958645e-3	.1369462854e-3

Para Gauss-Seidel, la componente i-ésima de la iteración (k+1)-ésima se calcula como:

$$x_i^{(k+1)} = \frac{1}{a_{ii}} \left(b_i - \sum_{j=1}^{i-1} a_{ij} x_j^{(k+1)} - \sum_{j=i+1}^{n} a_{ij} x_j^{(k)} \right)$$

para cada componente i = 1, 2, ... , n de cada iteración $\mathbf{x}^{(1)}$, $\mathbf{x}^{(2)}$, ..., cuando k = 0, 1, 2, ... Se utilizan las últimas componentes calculadas en la obtención de las siguientes.

GAUSS-SEIDEL						
Iter.	$\mathbf{x}^{(k)}$				Estimación errores	
k	$x_1^{(k)}$	$x_2^{(k)}$	$x_3^{(k)}$	$x_4^{(k)}$	$\|\| \mathbf{e}^{(k)} \|\|_\infty$	$\|\| \mathbf{e}^{(k)} \|\|_\infty / \|\| \mathbf{x}^{(k)} \|\|_\infty$
0	0	0	0	0	0	0
1	1.428571429	4.553571429	-3.383928572	7.517857143	7.517857143	1.000000000
2	-1.764030613	2.039540816	-1.328762755	6.679846938	3.192602042	.4779453888
3	-.5013210643	3.481858145	-2.365598806	7.160619382	1.442317329	.2014235434
4	-1.246064736	2.740742654	-1.808855143	6.913580885	.7447436717	.1077218426
5	-.8668137657	3.136113815	-2.101353466	7.045371272	.395371161	.5611786033e-1
6	-1.069781866	2.927931350	-1.946520208	6.975977117	.208182465	.2984276776e-1
7	-.9630260443	3.038046615	-2.028266796	7.012682205	.110115265	.1570230360e-1
8	-1.019514551	2.979893934	-1.985068329	6.993297978	.58152681e-1	.8315487368e-2
9	-.9896864843	3.010621484	-2.007889121	7.003540495	.30727550e-1	.4387430903e-2
10	-1.005448189	2.994388276	-1.995832091	6.998129425	.16233208e-1	.2319649583e-2
11	-.9971214900	3.002964758	-2.002202006	7.000988253	.8576482e-2	.1225038765e-2
12	-1.001520753	2.998433652	-1.998836638	6.999477883	.4531106e-2	.6473491417e-3
13	-.9991965514	3.000827532	-2.000614628	7.000275843	.2393880e-2	.3419693814e-3
14	-1.000424477	2.999562798	-1.999675280	6.999854267	.1264734e-2	.1806800473e-3
15	-.9997757400	3.000230982	-2.000171556	7.000076993	.668184e-3	.9545380725e-4
16	-1.000118480	2.999877968	-1.999909364	6.999959323	.353014e-3	.5043086448e-4
17	-.9999374043	3.000064472	-2.000047885	7.000021490	.186504e-3	.2664334678e-4
18	-1.000033070	2.999965939	-1.999974702	6.999988647	.98533e-4	.1407616569e-4
19	-.9999825286	3.000017995	-2.000013365	7.000005998	.52056e-4	.7436565056e-5
20	-1.000009231	2.999990494	-1.999992939	6.999996832	.27501e-4	.3928716064e-5

El método SOR (Successive Overrelaxation) pretende ser una aceleración del método de Gauss-Seidel. Introduce un parámetro ω de aceleración o relajación para realizar una media ponderada de la iteración anterior y la actual con Gauss-Seidel. Si ω = 1 es el método de Gauss-Seidel. En SOR, la componente i-ésima de la iteración (k+1)-ésima es por tanto:

$$x_i^{(k+1)} = \omega\, z_i^{(k+1)} + (1 - \omega)\, x_i^{(k)}$$

para cada componente i = 1, 2, ... , n de cada iteración $\mathbf{x}^{(1)}$, $\mathbf{x}^{(2)}$, ..., cuando k = 0, 1, 2, ... El valor $z_i^{(k+1)}$ es la i-ésima componente del vector de la (k+1)-ésima iteración calculada por Gauss-Seidel a partir del vector actual $\mathbf{x}^{(k+1)}$.

SOR, ω=.9						
Iter.	$\mathbf{x}^{(k)}$				Estimación errores	
k	$x_1^{(k)}$	$x_2^{(k)}$	$x_3^{(k)}$	$x_4^{(k)}$	$\|\| \mathbf{e}^{(k)} \|\|_\infty$	$\|\| \mathbf{e}^{(k)} \|\|_\infty / \|\| \mathbf{x}^{(k)} \|\|_\infty$
0	0	0	0	0	0	0
1	1.285714286	4.082142858	-2.801250000	6.624642858	6.624642858	1.000000000
2	-1.176681123	2.637634535	-1.877307288	6.853754646	2.462395409	.3592768542
3	-.8282805337	3.052096544	-2.049811054	7.001004428	.414462009	.5920036379e-1
4	-1.003345425	2.976814575	-1.993794943	6.993144816	.1750648913	.2503378607e-1

5	-.9883270215	3.002485012	-2.003124170	7.000059985	.25670437e-1	.3667173861e-2
6	-.9997167414	2.998523022	-1.999711510	6.999562906	.113897199e-1	.1627204449e-2
7	-.9991929800	3.000105367	-2.000200146	6.999987901	.1582345e-2	.2260496764e-3
8	-.9999461977	2.999904007	-1.999988923	6.999969992	.7532177e-3	.1076029898e-3
9	-.9999428184	3.000003065	-2.000013137	6.999997919	.99058e-4	.1415114706e-4
10	-.9999939012	2.999993604	-1.999999808	6.999997874	.510828e-4	.7297545074e-5
11	-.9999958531	2.999999935	-2.000000885	6.999999767	.6331e-5	.9044286015e-6
12	-.9999994079	2.999999562	-2.000000024	6.999999845	.35548e-5	.5078285827e-6
13	-.9999996927	2.999999977	-2.000000061	6.999999978	.415e-6	.5928571447e-7
14	-.9999999475	2.999999970	-2.000000004	6.999999990	.2548e-6	.3640000005e-7
15	-.9999999780	2.999999997	-2.000000004	6.999999997	.305e-7	.4357142859e-8
16	-.9999999952	2.999999998	-2.000000000	7.000000000	.172e-7	.2457142857e-8
17	-.9999999982	3.000000000	-2.000000000	7.000000000	.30e-8	.4285714286e-9
18	-.9999999998	3.000000000	-2.000000000	7.000000000	.16e-8	.2285714286e-9
19	-1.000000000	3.000000000	-2.000000000	7.000000000	.2e-9	.2857142857e-10
20	-1.000000000	3.000000000	-2.000000000	7.000000000	0.	0.

SOR, ω=1.1						
Iter.	$\mathbf{x}^{(k)}$				Estimación errores	
k	$x_1^{(k)}$	$x_2^{(k)}$	$x_3^{(k)}$	$x_4^{(k)}$	$\Vert \mathbf{e}^{(k)} \Vert_\infty$	$\Vert \mathbf{e}^{(k)} \Vert_\infty / \Vert \mathbf{x}^{(k)} \Vert_\infty$
0	0	0	0	0	0	0
1	1.571428571	5.028571429	-4.022857142	8.443809525	8.443809525	1.000000000
2	-2.441251701	1.208256463	-.4159111228	6.198646418	4.012680272	.6473478243
3	.1473627037	4.425997829	-3.258232438	7.603001230	3.217741366	.4232198929
4	-1.908112859	1.867499897	-1.001570663	6.524449839	2.558497932	.3921400264
5	-.2794982335	3.898739172	-2.792287464	7.377092711	2.031239275	.2753441436
6	-1.571727053	2.286815982	-1.371295104	6.700789922	1.611923190	.2405571893
7	-.5463179554	3.565934298	-2.498896918	7.237430251	1.279118316	.1767365310
8	-1.360010714	2.550913466	-1.604109768	6.811591912	1.015020832	.1490137467
9	-.7143203000	3.356364147	-2.314151221	7.149507662	.805450681	.1126582024
10	-1.226695732	2.717213959	-1.750711229	6.881361019	.639150188	.9288136260e-1
11	-.8201098830	3.224399525	-2.197818398	7.094143725	.507185566	.7149355661e-1
12	-1.142748407	2.821931992	-1.843024944	6.925294024	.402467533	.5811558782e-1
13	-.8867246953	3.141302506	-2.124564593	7.059281516	.319370514	.4524122083e-1
14	-1.089887479	2.887872064	-1.901154119	6.952958271	.253430442	.3644929714e-1
15	-.9286714881	3.088977008	-2.078437283	7.037329076	.201104944	.2857688504e-1
16	-1.056601396	2.929393974	-1.937757574	6.970378215	.159583034	.2289445839e-1
17	-.9550850314	3.056028079	-2.049391303	7.023505808	.126634105	.1803004204e-1
18	-1.035641427	2.955539974	-1.960806463	6.981347409	.100488105	.1439379809e-1
19	-.9717174193	3.035280415	-2.031101292	7.014801411	.79740441e-1	.1136745523e-1
20	-1.022443109	2.972003892	-1.975320157	6.988254619	.63276523e-1	.9054696265e-2

Mark summary: