Sistemas lineales

Question 1 Top 1 Bottom Focus Help

1.1

Obtener la factorización de Cholesky de la siguiente matriz (entrar sólo los elementos de U, la triangular superior)

$$\begin{bmatrix} 10 & -12 & -2 & -6 \\ -12 & 26 & -5 & 12 \\ -2 & -5 & 9 & 3 \\ -6 & 12 & 3 & 12 \end{bmatrix}$$

You have not attempted this yet

The teacher's answer was:

$$\begin{bmatrix} 10^{1/2} & -6/5\ 10^{1/2} & -1/5\ 10^{1/2} & -3/5\ 10^{1/2} \\ 0 & 1/5\ 290^{1/2} & -\frac{37}{290}\ 290^{1/2} & 12/145\ 290^{1/2} \\ 0 & 0 & 15/58\ 58^{1/2} & \frac{47}{145}\ 58^{1/2} \\ 0 & 0 & 0 & 2/5\ \sqrt{2} \end{bmatrix}$$

1.2

Entrar el valor del determinante:
You have not attempted this yet

The teacher's answer was:

144

1.3

Resolver el sistema lineal A**x**=**b** cuando **b** es el vector siguiente

$$\begin{bmatrix} 14 \\ -35 \\ 1 \\ -27 \end{bmatrix}$$

You have not attempted this yet

The teacher's answer was:

$$\begin{bmatrix} 2 & 2 & 3 & -4 \end{bmatrix}$$

Solution:

Factorización

En cada etapa de la resolución se muestran los valores actuales de la matriz. Los nuevos elementos calculados aparecen con su valor definitivo en color diferente.

Calculando el elemento (1,1)

$$\begin{bmatrix} \texttt{10^(1/2)} & -12 & -2 & -6 \\ -12 & 26 & -5 & 12 \\ -2 & -5 & 9 & 3 \\ -6 & 12 & 3 & 12 \end{bmatrix}$$

Tratando la fila/columna 1

$$\begin{bmatrix} \texttt{10^(1/2)} & \texttt{-6/5*10^(1/2)} & \texttt{-1/5*10^(1/2)} & \texttt{-3/5*10^(1/2)} \\ \texttt{-6/5*10^(1/2)} & 26 & -5 & 12 \\ \texttt{-1/5*10^(1/2)} & -5 & 9 & 3 \\ \texttt{-3/5*10^(1/2)} & 12 & 3 & 12 \end{bmatrix}$$

Calculando el elemento (2,2)

$$\begin{bmatrix} \texttt{10^(1/2)} & \texttt{-6/5*10^(1/2)} & \texttt{-1/5*10^(1/2)} & \texttt{-3/5*10^(1/2)} \\ \texttt{-6/5*10^(1/2)} & \texttt{1/5*290^(1/2)} & -5 & 12 \\ \texttt{-1/5*10^(1/2)} & -5 & 9 & 3 \\ \texttt{-3/5*10^(1/2)} & 12 & 3 & 12 \end{bmatrix}$$

Tratando la fila/columna 2

$$\begin{bmatrix} \texttt{10^(1/2)} & \texttt{-6/5*10^(1/2)} & \texttt{-1/5*10^(1/2)} & \texttt{-3/5*10^(1/2)} \\ \texttt{-6/5*10^(1/2)} & \texttt{1/5*290^(1/2)} & \texttt{-37/290*290^(1/2)} & \texttt{12/145*290^(1/2)} \\ \texttt{-1/5*10^(1/2)} & \texttt{-37/290*290^(1/2)} & 9 & 3 \\ \texttt{-3/5*10^(1/2)} & \texttt{12/145*290^(1/2)} & 3 & 12 \end{bmatrix}$$

Calculando el elemento (3,3)

$$\begin{bmatrix} 10\text{^}(1/2) & -6/5*10\text{^}(1/2) & -1/5*10\text{^}(1/2) & -3/5*10\text{^}(1/2) \\ -6/5*10\text{^}(1/2) & 1/5*290\text{^}(1/2) & -37/290*290\text{^}(1/2) & 12/145*290\text{^}(1/2) \\ -1/5*10\text{^}(1/2) & -37/290*290\text{^}(1/2) & 15/58*58\text{^}(1/2) & 3 \\ -3/5*10\text{^}(1/2) & 12/145*290\text{^}(1/2) & 3 & 12 \end{bmatrix}$$

Tratando la fila/columna 3

$$\begin{bmatrix} 10\text{^}(1/2) & -6/5*10\text{^}(1/2) & -1/5*10\text{^}(1/2) & -3/5*10\text{^}(1/2) \\ -6/5*10\text{^}(1/2) & 1/5*290\text{^}(1/2) & -37/290*290\text{^}(1/2) & 12/145*290\text{^}(1/2) \\ -1/5*10\text{^}(1/2) & -37/290*290\text{^}(1/2) & 15/58*58\text{^}(1/2) & 47/145*58\text{^}(1/2) \\ -3/5*10\text{^}(1/2) & 12/145*290\text{^}(1/2) & 47/145*58\text{^}(1/2) & 12 \end{bmatrix}$$

Calculando el elemento (4,4)

$$\begin{bmatrix} 10\text{^}(1/2) & -6/5*10\text{^}(1/2) & -1/5*10\text{^}(1/2) & -3/5*10\text{^}(1/2) \\ -6/5*10\text{^}(1/2) & 1/5*290\text{^}(1/2) & -37/290*290\text{^}(1/2) & 12/145*290\text{^}(1/2) \\ -1/5*10\text{^}(1/2) & -37/290*290\text{^}(1/2) & 15/58*58\text{^}(1/2) & 47/145*58\text{^}(1/2) \\ -3/5*10\text{^}(1/2) & 12/145*290\text{^}(1/2) & 47/145*58\text{^}(1/2) & 2/5*2\text{^}(1/2) \end{bmatrix}$$

La factorización final es la siguiente, en la que aparecen las matrices U^T y U, y el vector de permutaciones:

$$\left[\begin{bmatrix} 10^{1/2} & 0 & 0 & 0 \\ -6/5\ 10^{1/2} & 1/5\ 290^{1/2} & 0 & 0 \\ -1/5\ 10^{1/2} & -\frac{37}{290} 290^{1/2} & 15/58\ 58^{1/2} & 0 \\ -3/5\ 10^{1/2} & 12/145\ 290^{1/2} & \frac{47}{145} 58^{1/2} & 2/5\ \sqrt{2} \end{bmatrix}, \begin{bmatrix} 10^{1/2} & -6/5\ 10^{1/2} & -1/5\ 10^{1/2} & -3/5\ 10^{1/2} \\ 0 & 1/5\ 290^{1/2} & -\frac{37}{290} 290^{1/2} & 12/145\ 290^{1/2} \\ 0 & 0 & 15/58\ 58^{1/2} & \frac{47}{145} 58^{1/2} \\ 0 & 0 & 0 & 2/5\ \sqrt{2} \end{bmatrix} \right]$$

Determinante

El valor del determinante viene dado por el producto de los elementos de la diagonal principal de U y coincide con la diagonal principal de U^T. Por tanto, es

$$|A| = |U^T\ U| = |U^T|\ |U| = |U|^2 = \prod_{i=1}^{4} u_{ii}^2 = 144$$

Resolución del sistema

Queremos resolver A x = b ⇒ U^T U x = b. Llamando y = U x, como en la factorización LU (Crout y Doolittle), podemos resolver el sistema en dos pasos:

[1] $U^T y = b$, de donde se obtiene el vector y,
[2] $U x = y$, de donde ya se puede obtener el vector solución x.

Resolvemos entonces el primer sistema triangular

$$\begin{bmatrix} 10^{1/2} & 0 & 0 & 0 \\ -6/5\ 10^{1/2} & 1/5\ 290^{1/2} & 0 & 0 \\ -1/5\ 10^{1/2} & -\frac{37}{290}\ 290^{1/2} & 15/58\ 58^{1/2} & 0 \\ -3/5\ 10^{1/2} & 12/145\ 290^{1/2} & \frac{47}{145}\ 58^{1/2} & 2/5\sqrt{2} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} 14 \\ -35 \\ 1 \\ -27 \end{bmatrix} \Rightarrow \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} 7/5\ 10^{1/2} \\ -\frac{91}{290}\ 290^{1/2} \\ -\frac{151}{290}\ 58^{1/2} \\ -8/5\sqrt{2} \end{bmatrix}$$

por sustitución hacia adelante ($y_1 \Rightarrow y_2 \Rightarrow y_3 \Rightarrow y_4$). Resolvemos ahora

$$\begin{bmatrix} 10^{1/2} & -6/5\ 10^{1/2} & -1/5\ 10^{1/2} & -3/5\ 10^{1/2} \\ 0 & 1/5\ 290^{1/2} & -\frac{37}{290}\ 290^{1/2} & 12/145\ 290^{1/2} \\ 0 & 0 & 15/58\ 58^{1/2} & \frac{47}{145}\ 58^{1/2} \\ 0 & 0 & 0 & 2/5\sqrt{2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 7/5\ 10^{1/2} \\ -\frac{91}{290}\ 290^{1/2} \\ -\frac{151}{290}\ 58^{1/2} \\ -8/5\sqrt{2} \end{bmatrix} \Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 3 \\ -4 \end{bmatrix}$$

por sustitución hacia atrás ($x_4 \Rightarrow x_3 \Rightarrow x_2 \Rightarrow x_1$), resultando el vector pedido.

Mark summary:

Question	Value	Your mark
1.1	1.50	-
1.2	0.50	-
1.3	1.00	-
Total	3.00	0.00

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